Grade 10 Trigonometry Concept Map

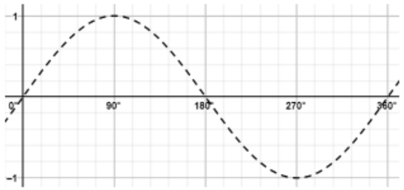
Trigonometry

θ ; α ; β

Pythag; Rounding; Equations

Functions

$y = \sin x$



Calculator work

$\sin 40° = m$

vs

$\sin \theta = 0,5$

Trig ratios

$\text{Sin}\,\theta = \frac{O}{H}$

$\text{Cos}\,\theta = \frac{A}{H}$

$\text{Tan}\,\theta = \frac{O}{A}$

Reciprocals

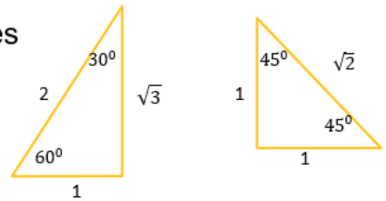
$\text{Cot}\,\theta = \frac{1}{Tan\theta}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

$\text{Sec}\,\theta = \frac{1}{\text{Cos}\,\theta}$

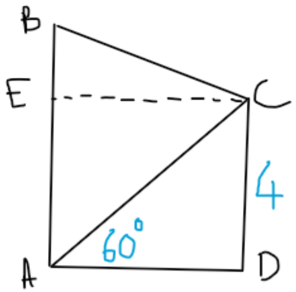
Without a calculator

- Special angles



- Diagram

2D problems



Unit circle/ CAST diagram

Trig equations

Solve for x:

$2\cos(x + 10) = 0,6$

Trigonometry Notes

Scan here with your phone to access videos exploring these concepts.

Notation

The angles of a triangle are often named using Greek letters:

θ (theta), α (alpha), β (beta)

They just represent unknown angles.

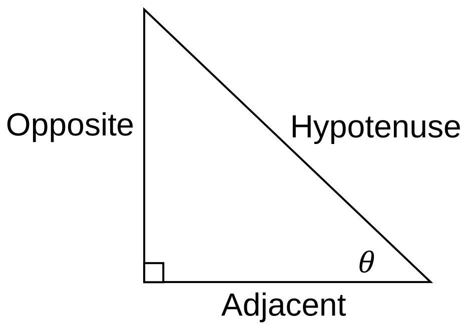
Trig Ratios

In a right-angled triangle:

sine $\theta = \frac{\text{opp}}{\text{hyp}}$ SOH

cosine $\theta = \frac{\text{adj}}{\text{hyp}}$ CAH

tangent $\theta = \frac{\text{opp}}{\text{adj}}$ TOA



Using a Calculator

1. To find the values of ratios (angle given)

Example: $sin 50°$

Key in the ratio (Sin), then the angle (50), then press = .

$$\therefore sin 50° = 0,766 \ldots$$

2. To calculate an angle (ratio of sides given)

Example: $sin\theta° = \frac{1}{3}$

Key in: Shift, then the ratio (it gives you this: $\sin^{-1)}$, then the ration ($\frac{1}{3}$), then press = .

$$\therefore \theta = 19{,}47°$$

Trig Equations

- First, isolate the trig ratio.
- Make sure you know how to use your calculator

<u>Solve for θ:</u>

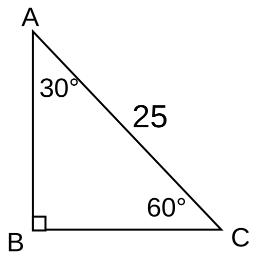
1. $sin\,\theta = 0{,}5$ $\quad\quad \therefore\ \theta = 30°$
2. $2\,cos\,\theta = 0{,}8$ $\quad\quad \therefore cos\,\theta = 0{,}4$ $\quad\quad \therefore\ \theta = 66{,}4°$
3. $3\,tan\,\theta - 1 = 2$ $\quad\quad \therefore 3\,tan\,\theta = 3$ $\quad\quad \therefore tan\,\theta = 1$ $\quad\quad \theta = 45°$
4. $sin(\theta - 15°) - 1 = 0$ $\quad\quad \therefore sin(\theta - 15°) = 1$

$$\therefore\ \theta - 15° = 90°$$

$$\therefore\ \theta = 90° + 15° = 105°$$

Finding side lengths (given the Angle)

1. Determine the length of AB



$$\sin 60° = \frac{AB}{25}$$
$$25 \sin 60° = AB$$
$$21{,}7 = AB$$

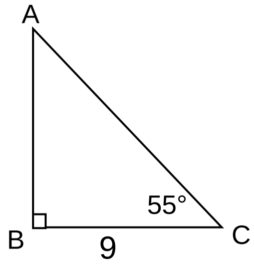
or

$$\cos 30° = \frac{AB}{25}$$
$$25 \cos 30° = AB$$
$$21{,}7 = AB$$

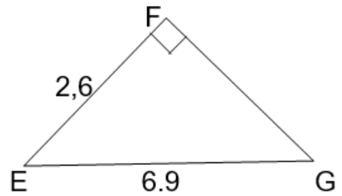
2. Determine the length of AB and AC

$$\tan 55° = \frac{AB}{9}$$
$$9\tan 55° = AB$$
$$12{,}9 = AB$$

$$\cos 55° = \frac{9}{AC}$$
$$AC\cos 55° = 9$$
$$AC = \frac{9}{\cos 55°}$$
$$AC = 15{,}7$$



Finding an Angle (given the sides)

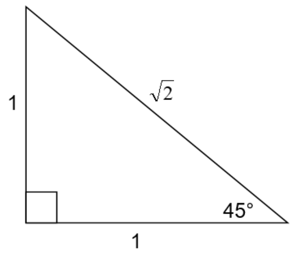


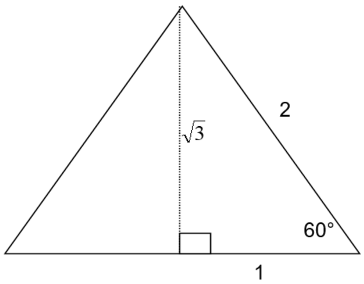
Calculate Ê

$$\frac{EF}{EG} = \cos \hat{E}$$
$$\cos \hat{E} = \frac{2{,}6}{6{,}9}$$
$= 0{,}3768\ldots$ (don't round off yet, but use $\cos^{-1}$)
$$\therefore \hat{E} = 67{,}86°$$

Special Angles

- These angles are 0º, 30º, 45º, 60º and 90º.
- Use the triangles when you are asked not to use your calculator to evaluate trig ratios.





Example: $tan\ 45° + (tan\ 30°)(tan\ 60°)$

$$= 1 + \left(\frac{\sqrt{3}}{3}\right)\left(\frac{\sqrt{3}}{1}\right)$$

$$= 1 + \left(\frac{3}{3}\right)$$

$$= 2$$

Cartesian Plane (CAST Diagram)

Trig ratios $0° \leq \theta \leq 360°$

- The question will state ‘without using a calculator’ or ‘with the aid of a diagram’
- You will be given a trig ratio
- You will be asked to determine the value of other ratios

How-to

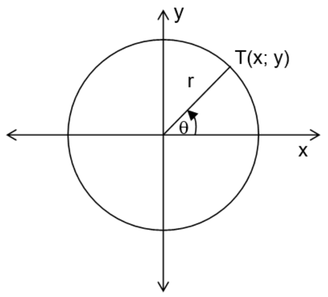
- Arrange the equation so that the trig ratio is by itself
- Determine in which quarter the angle is – use the given ratio and the restrictions
- Determine which values (x, y or r) are given, and which values must be calculated
- Use Pythagoras’ theorem to calculate the unknown values

Take Note

- r is always positive
- write down the values of x, y and r and determine the sign of x and y in the relevant quadrants

Quadrant 1

$\theta \in [0°; 90°]$

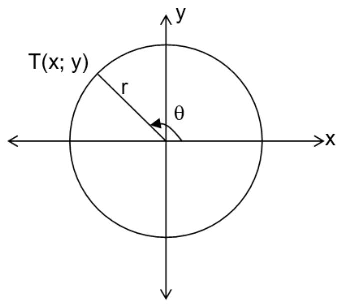


x and y are positive

$\therefore$ sinθ, cosθ and tanθ are all positive

Quadrant 2

$\theta \in [90°; 180°]$

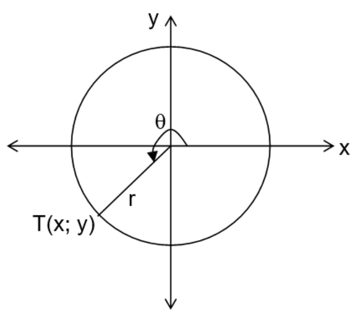


x is negative and y is positive

$\therefore$ sinθ is positive and cosθ and tanθ are negative.

Quadrant 3

$\theta \in [180°; 270°]$

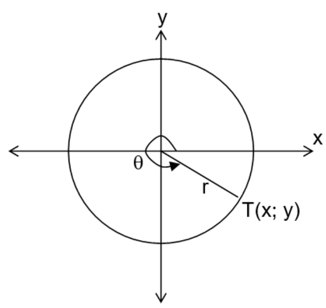


x and y are negative

$\therefore$ tan θ is positive and sin θ and cosθ are negative.

Quadrant 4

$\theta \in [270°; 360°]$

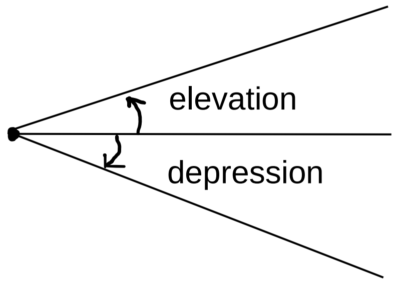


x is positive and y is negative

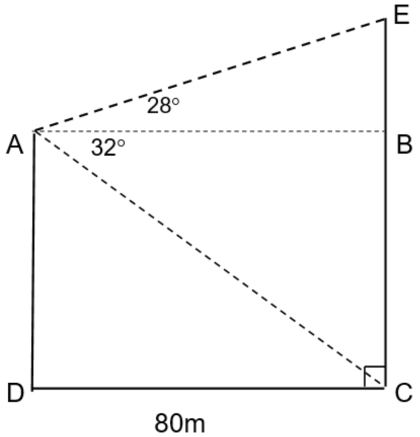
$\therefore$ cos θ is positive and sin θ and tanθ are negative.

Practical Application

Trigonometry enables us to calculate heights and angles that we would not be able to reach otherwise. We use our knowledge on trig ratios in right angles triangles to find missing information, angles and/or sides.



Example:



Thandi is at Point A, a lookout point with a telescope on top of a building. She looks up at a Star at E, through an angle of elevation of 28°. She looks at point C, her parked car at an angle of depression of 32° from A.
Calculate the height of the star directly above the car, if the car is 80m from the foot of the building.

$$\Delta ACB:$$
$$AB = 80m$$
$$\tan 32^0 = \frac{CB}{80}$$
$$80 \times \tan 32^0 = CB$$
$$\therefore CB = 50\ m$$

$$\Delta ABE:$$
$$\tan 28^0 = \frac{EB}{80}$$
$$80 \times \tan 28^0 = CB$$
$$\therefore CB = 42{,}5\ m$$

$$\therefore 50\ m + 42{,}5\ m = 92{,}5\ m$$

Trig Functions

Trigonometric functions model real-life situations involving waves, cycles, and rotations, like sound waves, tides, and the motion of planets. Understanding how these functions behave (such as vertical stretches and shifts) helps us analyse patterns in science, engineering, and various fields.

Amplitude refers to the maximum distance the graph of a trigonometric function moves away from its central axis (usually the $x - axis$). It measures the height of the wave from the middle line to the maximum.

A vertical stretch (a) occurs when the amplitude of a trigonometric function increases or decreases.

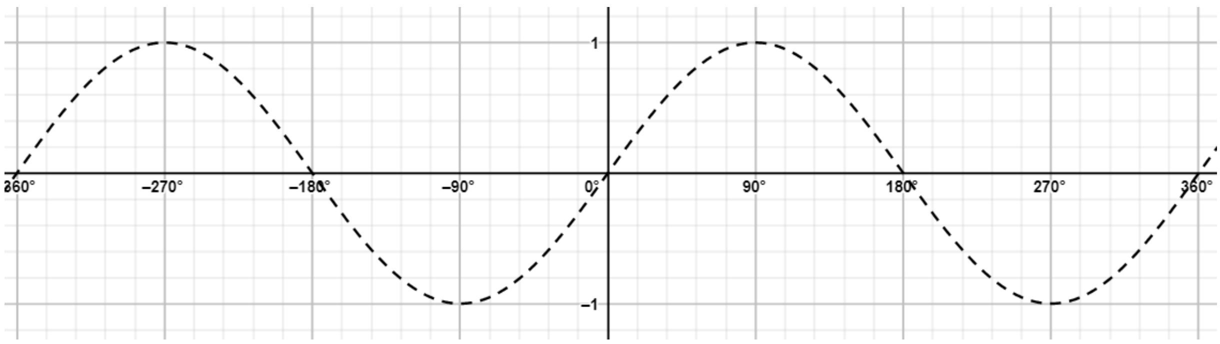
Calculate the a value: $\frac{max - min}{2}$

A vertical shift (q) moves the entire graph of a trigonometric function up or down.

Sine Graph

$$y = a\sin(x) + q$$

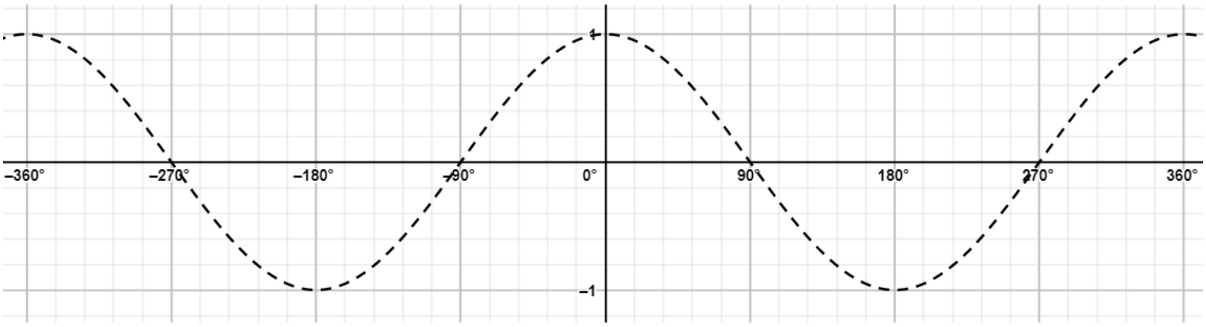
- Period: 360°
- a: the amplitude (a may sometimes be negative, but the amplitude must be given as a positive value)
- q: vertical shift (moves the graph up or down)



Cosine Graph

$$y = a\cos(x) + q$$

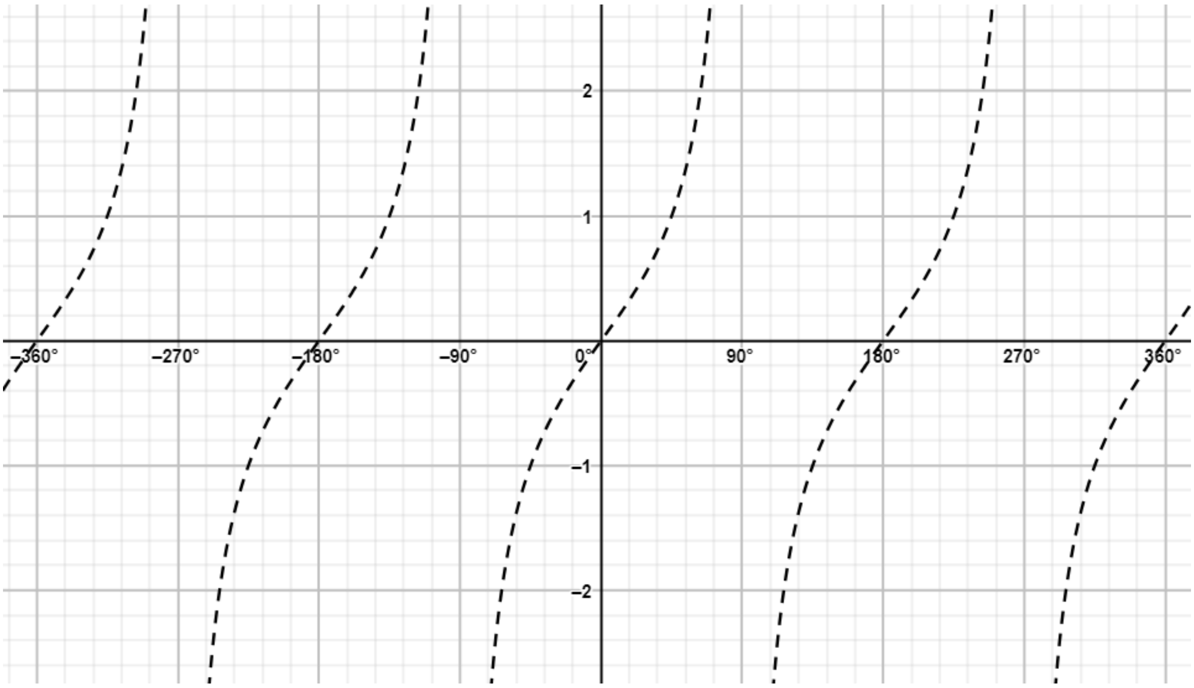
- Period: 360°
- *a*: the amplitude (a may sometimes be negative, but the amplitude must be given as a positive value)
- *q*: vertical shift



Tangent Graph

$$y = a\tan(x) + q$$

- Period: 180°
- a: the y value that corresponds to 45° on the $x - axis$
- q: vertical shift
- Asymptotes: at - 90°, 90°, - 270°, 270° etc

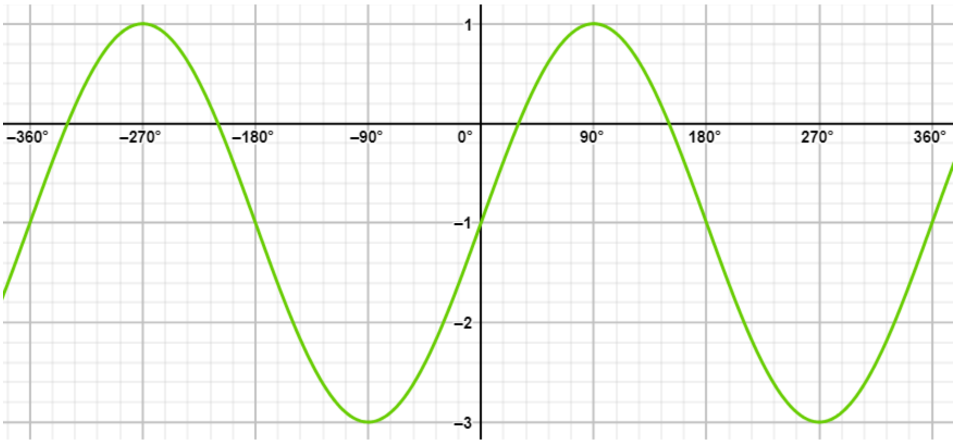


Finding the Equation

It helps when you know what your basic functions look like: $y = sinx$, $y = cosx$ and $y = tanx$

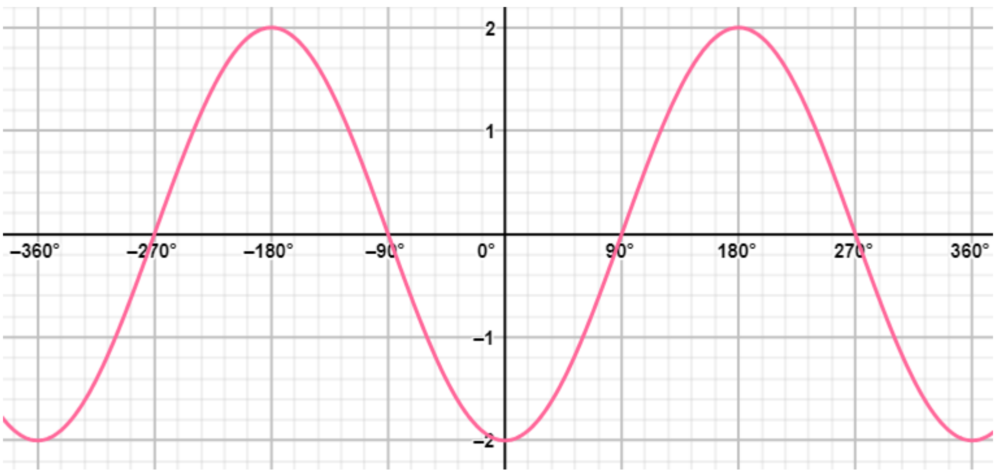
Compare the transformed graph with the basic graph to determine how it changed.

<u>Examples:</u>
Find the equation, domain and range of the following graphs:



- The graph moved down (the middle of the graph is not the x-axis anymore)
- Graph goes through $(0°; -1)$ instead of $(0°; 0)$, ∴ moved down 1 unit
- a value: 2

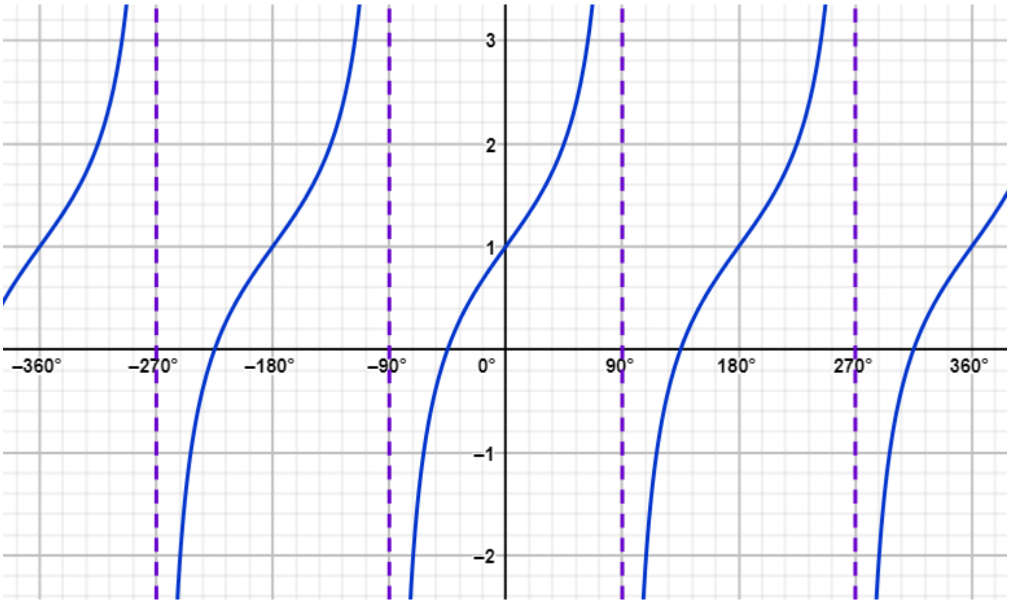
$\frac{max-min}{2} = \frac{1-(-3)}{2} = 2$ $\therefore y = 2sinx - 1$



- The graph did not move up or down (the middle of the graph is still the x-axis)
- a value: -2

$\therefore y = -2cosx$

The a value must be a negative. If we compare the graph with the basic graph: $y = cosx$, we can see that the graph was reflected about the x-axis.



- The graph moved up (the basic tan graph, $y = tanx,$ goes through $(0°, 0)$, this graph goes through $(0°, 1)$, $\therefore$ the graph moved 1 unit up)
- a value: 1 (the graph goes through $(45°, 2)$, the basic tan graph does through $(45°, 1)$ and since this graph moved 1 up, the a didn't change)

$\therefore y = tan + 1$

Need help?